You are trying to find the volume of a solid using *cyclindrical shells*.
The solid was formed by rotation around the x-axis.

What is the radius of a shell: x y either

Your integral and bounds should be with respect to: x y either

You are trying to find the volume of a solid using the *disc method*.
The solid was formed by rotation around the y-axis.

What is the radius of a disc: x y either

Your integral and bounds should be with respect to: x y either

You are trying to find the volume of a solid using the *disc method*.
The solid was formed by rotation around the x-axis.

What is the radius of a disc: x y either

Your integral and bounds should be with respect to: x y either

You are trying to find the *area* of a surface formed by rotation around the y-axis

What is the radius of a band: x y either

Your integral and bounds should be with respect to: x y either

You are trying to find the *area* of a surface formed by rotation around the x-axis

What is the radius of a band: x y either

Your integral and bounds should be with respect to: x y either

1)Find the length of $y = \frac{1}{2}\int_2^{x^2} \sqrt{t-2}\,dt$ from $x = \sqrt{2}$ to $x = \sqrt{3}$

By the fundamental theorem of calculus and the chain rule $\frac{dy}{dx} = x\sqrt{x^2-2}$

$$
\begin{aligned}
L &= \int_{\sqrt{2}}^{\sqrt{3}} \sqrt{1+(x\sqrt{x^2-2})^2}\,dx \\
&= \int_{\sqrt{2}}^{\sqrt{3}} \sqrt{1+x^2(x^2-2)}\,dx \\
&= \int_{\sqrt{2}}^{\sqrt{3}} \sqrt{1+x^4-2x^2}\,dx \\
&= \int_{\sqrt{2}}^{\sqrt{3}} \sqrt{(x^2-1)^2}\,dx \\
&= \int_{\sqrt{2}}^{\sqrt{3}} x^2-1\,dx \\
&= [\frac{x^3}{3} - x]_{\sqrt{2}}^{\sqrt{3}} \\
&= \frac{3\sqrt{3}}{3} - \sqrt{3} - (\frac{2\sqrt{2}}{3} - \sqrt{2})
\end{aligned}
$$

2)Find α so that the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = \alpha$ is $\frac{56}{27}$

$$
\begin{aligned}
L &= \int_0^\alpha \sqrt{1+(\frac{dy}{dx})^2}\,dx \\
&= \int_0^\alpha \sqrt{1+(\frac{3\sqrt{x}}{2})^2}\,dx \\
&= \int_0^\alpha \sqrt{1+\frac{9x}{4}}\,dx \\
&= [\frac{2}{3}\cdot(1+\frac{9x}{4})^{3/2}\cdot\frac{4}{9}]_0^\alpha \\
&= \frac{8}{27}[(1+\frac{9\alpha}{4})^{3/2}-1]
\end{aligned}
$$

If $L = \frac{56}{27}$, then we can solve the above equation for α:

$\frac{56}{27} = \frac{8}{27}[(1+\frac{9\alpha}{4})^{3/2}-1]$

$7 = (1+\frac{9\alpha}{4})^{3/2}-1$
$8^{2/3} = 1+\frac{9\alpha}{4}$
$4-1 = \frac{9\alpha}{4}$
$\alpha = \frac{4}{3}$

3)If two parallel planes intersect a sphere, show that the surface area of the part of the sphere lying between the two planes depends only on the radius of the sphere and the distance between the planes, and not on the position of the planes.

$$\int_a^b 2\pi\sqrt{r^2-x^2}\sqrt{1+\frac{x^2}{r^2-x^2}}\,dx = \int_a^b r\,dx = r(b-a)$$